

CASE STUDY: BEHAVIOUR MODELLING OF Ti6-4 ALLOY AT HIGH TEMPERATURE

3. Identification procedure of the model parameters

3.1 Identification of the elastic properties E and σ_0

1. Description of the test:

A tensile test is performed at different temperatures, divided into two different stages. First, constant strain rate is applied until a final deformation of $1E-2$ is attained. Beyond this point, the strain remains constant until the end of the test (not plotted to aid visualisation of the linear stage).

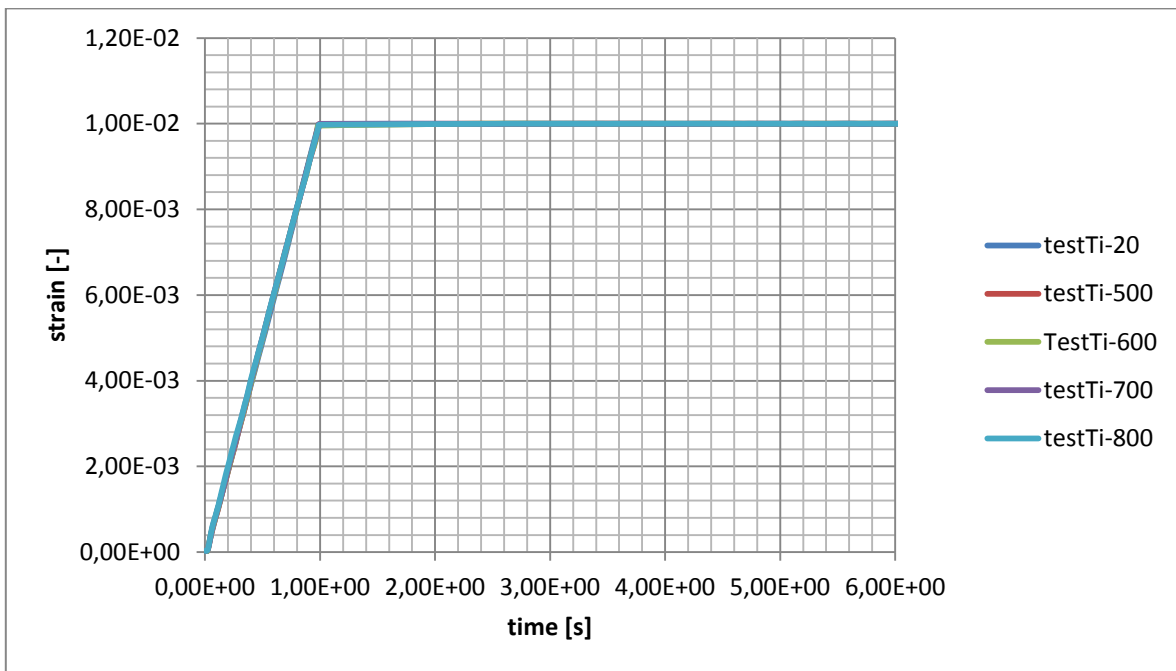


Figure 1 - Strain vs. time

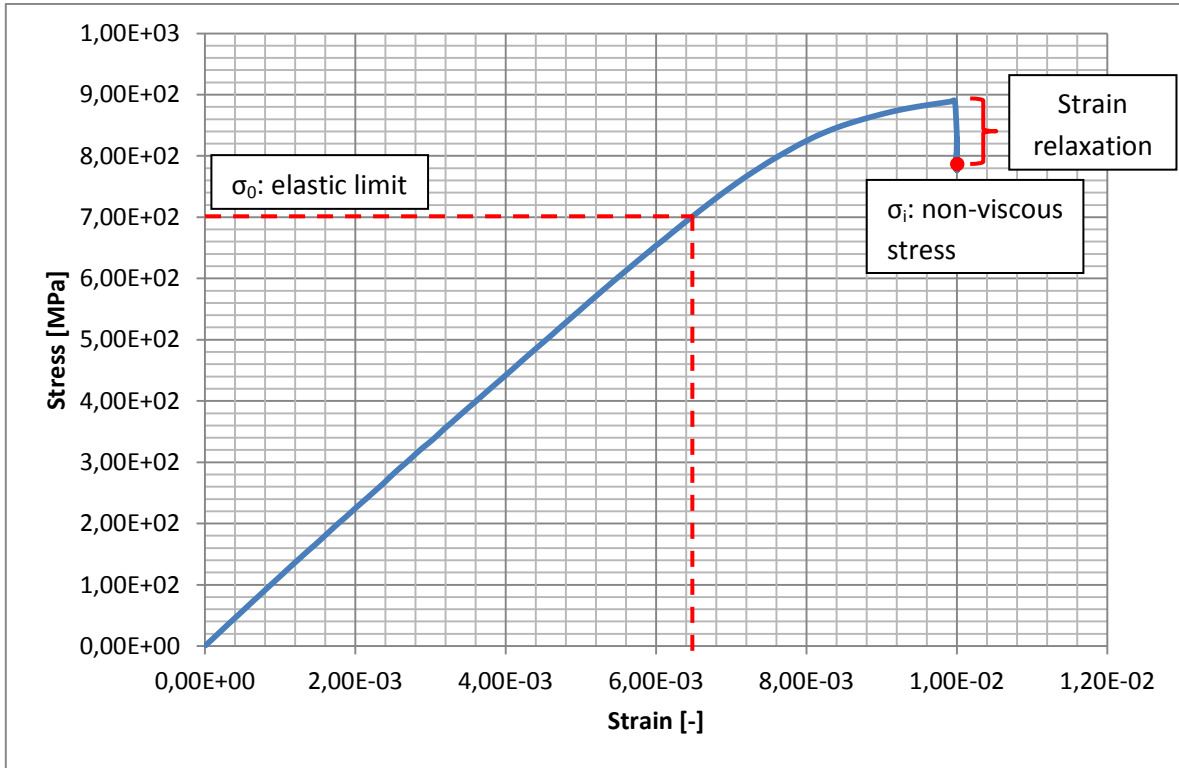


Figure 2 - Schematic stress vs. strain

In the stress vs. strain curves it can be observed that as the strain is increased linearly with time, the stress also increases, for all temperatures. Firstly, there is the linear, elastic stage, then the plastic region, where the slope continually decreases, and finally the maximum stress for each test is reached, when maximum strain is attained. At this point, strain is maintained constant and measurements of the stresses show a decrease, called stress relaxation. This relaxation is due to the disappearance of viscous effects dependant on strain rate, as strain is kept constant.

It can, then, be observed that beyond the elastic limit, two effects have to be overcome in order to increase stresses. A strain hardening, non-viscous effect, depending on strain; and a viscous effect, depending on strain rate.

The test is finished when an asymptotic value of stress (σ_i : non-viscous stress) is reached. This value allows to quantify the viscous effects (depending on strain rate), considering the difference of the actual stress at any given moment, and the final, asymptotic non-viscous stress.

$$\sigma - \sigma_i = K \dot{\epsilon}_{in}^{1/n}$$

The evolution of non-viscous stress can be approximated by the following relation:

$$R = Q(1 - e^{-b\epsilon_{in}})$$

And a linear relation between these two effects can be defined as:

$$\sigma_i = \sigma_0 + R$$

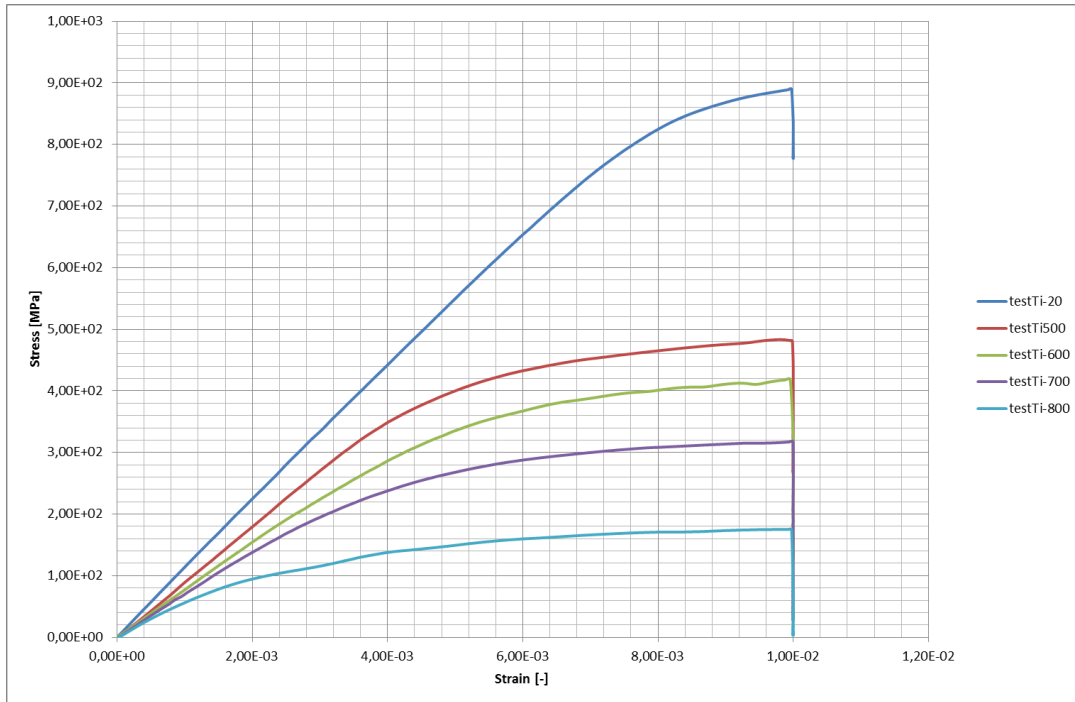


Figure 3 - Stress vs. Strain

A noteworthy observation is that as temperature rises, viscous effects become dominant over non-viscous effects. At 800 °C, for example, there is nearly no strain hardening effect, but all the stress needed is to overcome viscous effects. As non-viscous effects are not present, a much lower stress is necessary to achieve the same strain levels than at lower temperatures.

Another relevant observation is that the higher the temperature, the lower the springback effect due to elastic recovery. In fact, at 800 °C, this effect is negligible.

The Young Modulus for decreases with temperature and can be calculated as the slope of the initial linear part.

To do so, elastic limit is determined by inspection. Then all the points in the curve corresponding to stresses below the defined elastic limit are plotted against strain. This way, an approximately linear relation is obtained.

To estimate the slope of the curve a minimum squares regression is used, imposing that the curve passes through the origin. It is verified that dispersion is small to validate the use of this approximation. The following results are obtained.

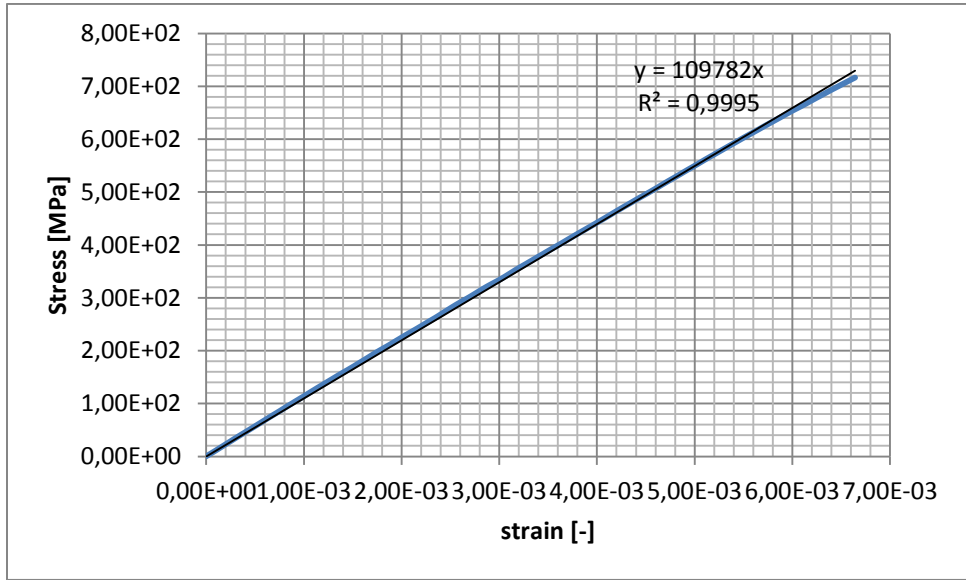


Figure 4 - E testTi-20

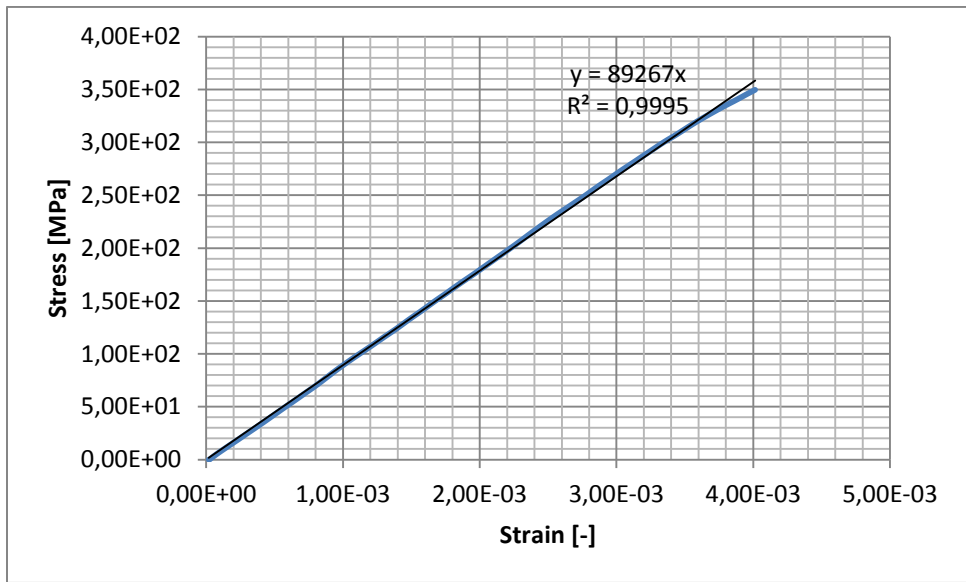


Figure 5 - E testTi-500

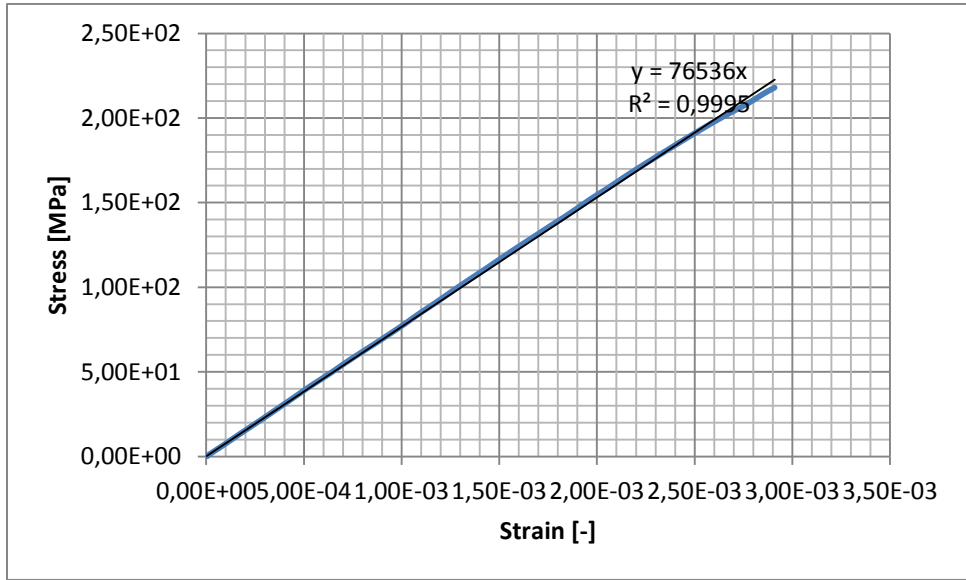


Figure 6 - E testTi-600

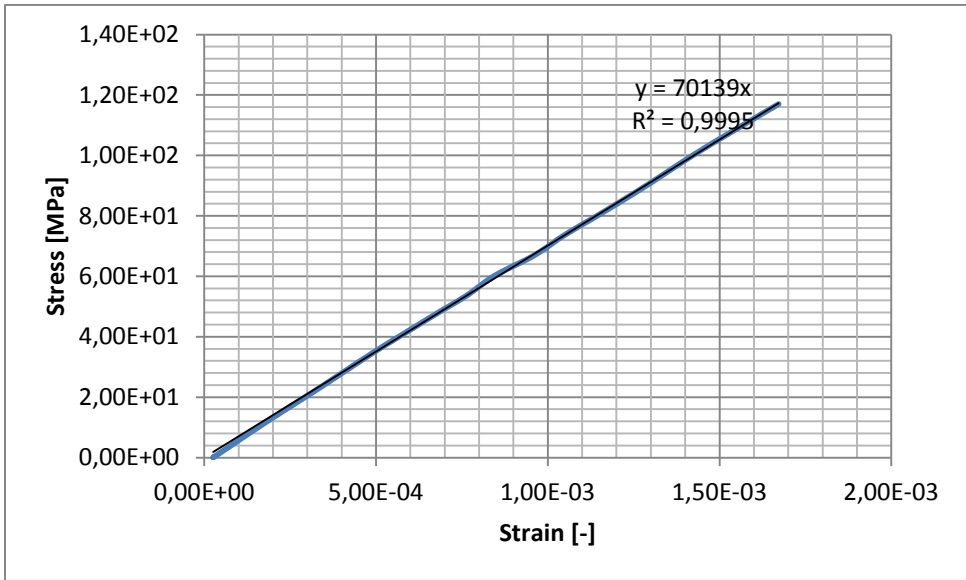


Figure 7 - E testTi-700

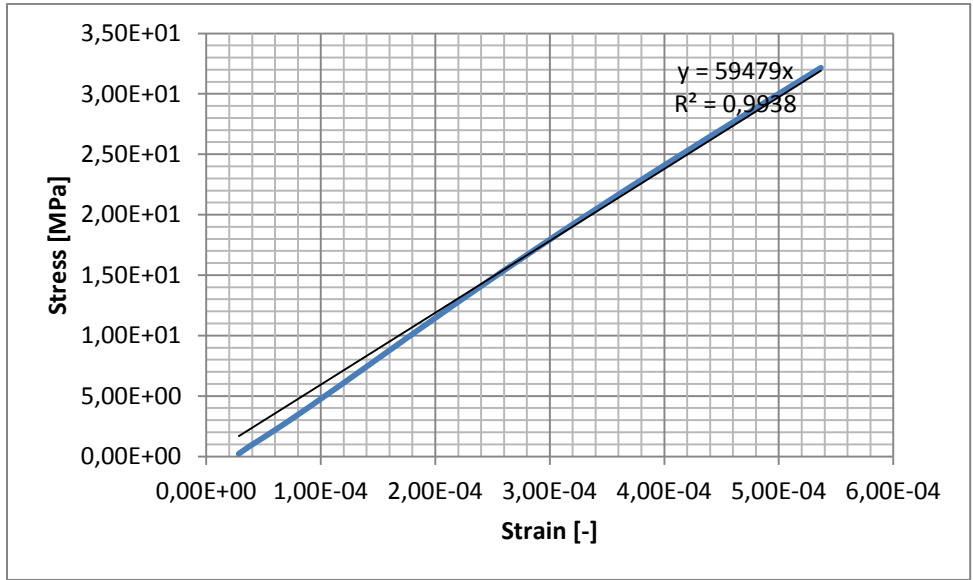


Figure 8- E testTi-800

As it was previously stated, the stiffness decreases as the test temperature rises.

E (20°C)	E (500°C)	E(600°C)	E(700°C)	E(800°C)
109782 MPa	89267 MPa	76536 MPa	70139 MPa	59479 MPa

Table 1- Young Modulus variation with temperature

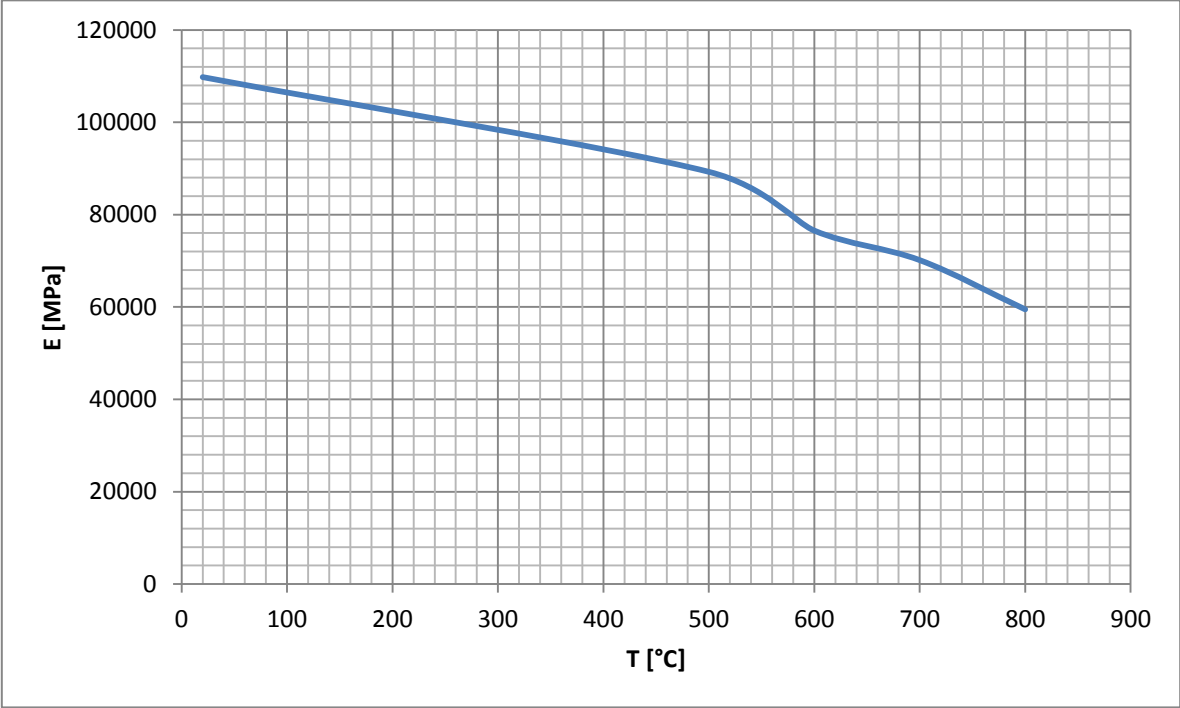


Figure 9 - Young modulus variation with temperature

2. The elasticity limit was obtained by inspection of Figure 3 (stress vs. strain). It was approximated searching the higher value around that region without decreasing R^2 .

This value is not simple to obtain, since it is very difficult to determine when the slope has decreased significantly to assume there is no more elasticity.

Just as E, yield stress decreases with increase of test temperature.

σ_0 (20°C)	σ_0 (500°C)	σ_0 (600°C)	σ_0 (700°C)	σ_0 (800°C)
7.16E2 MPa	3.5E2 MPa	2.18E2 MPa	1.17E2 MPa	3.22E1 MPa

Table 2- Young Modulus variation with temperature

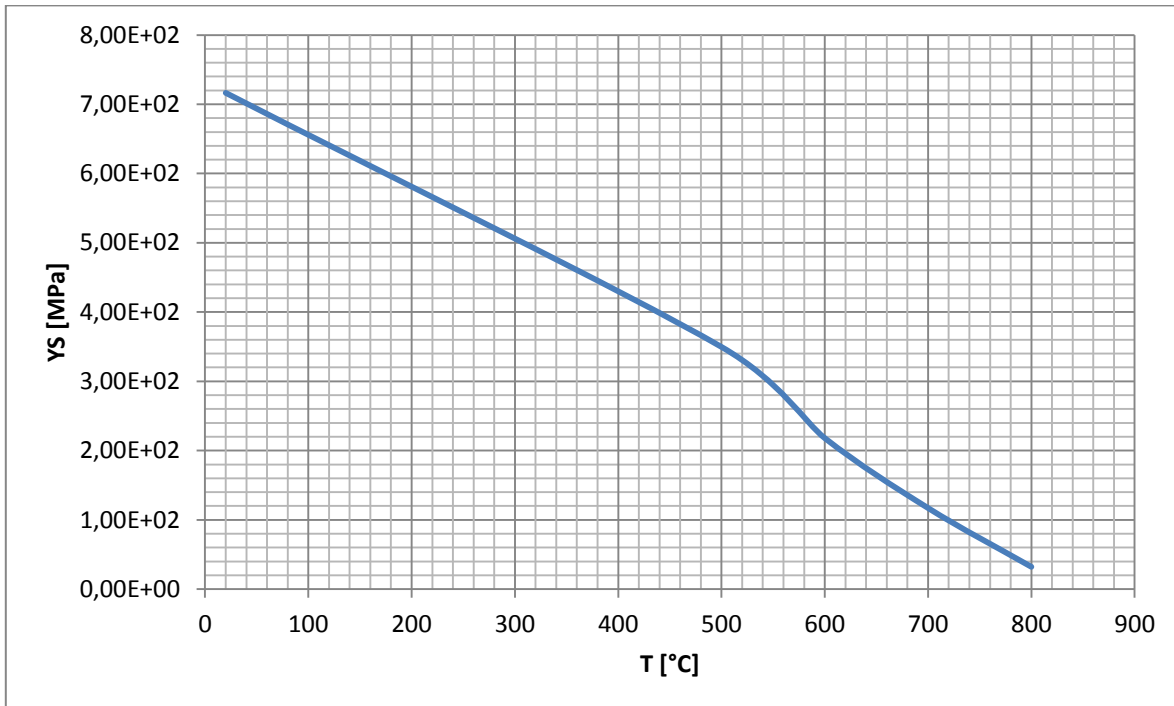


Figure 10 – Elastic limit variation with temperature

3. The asymptotic values of stresses after dwell time provide the residual stresses due to initial plastic hardening.

High temperature proves advantageous to reduce these effects and decrease final stress required to achieve a given final strain.

σ_i (20°C)	σ_i (500°C)	σ_i (600°C)	σ_i (700°C)	σ_i (800°C)
7.77E2 MPa	2.68E2 MPa	1.29E2 MPa	2.96E1 MPa	4.78 MPa

Table 3 - Non viscous stresses σ_i

4. Given the strain rate sensitivity and the normality rule:

$$\left. \begin{array}{l} |\dot{\varepsilon}_{in}| = \dot{p} \\ \dot{p} = \left(\frac{f}{K}\right)^n \end{array} \right\} f = K \dot{\varepsilon}_{in}^{1/n}$$

Considering the Von Mises yield criterion and the viscous stress expression:

$$\left. \begin{array}{l} f = |\sigma| - R - \sigma_0 \\ \sigma_i = \sigma_0 + R \end{array} \right\} f = \sigma - \sigma_i$$

So finally we have the relation:

$$\boxed{\sigma - \sigma_i = K \dot{\varepsilon}_{in}^{1/n}}$$

5. As the strain remains constant, a decrease in stress can be observed, due to the attenuation of viscous effects dependant on stress rate.

A final, asymptotic value is reached, once viscous effects have completely disappeared, after dwell time. This value corresponds to final non-viscous stress σ_i .

As it was also observed in stress vs. strain curves, stress relaxation increases with temperature, and non-viscous stresses decrease, up to the limit case at 800 °C, where non viscous stress approaches 0.

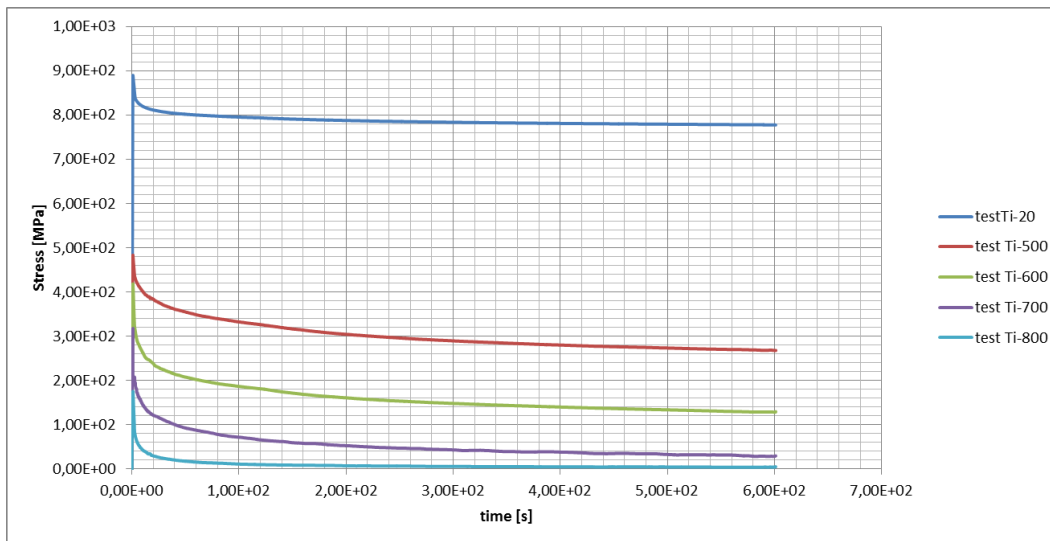


Figure 11 - Stress relaxation

6. Given:

$$\varepsilon_{tot} = \varepsilon_{el} + \varepsilon_{in}$$

Elastic deformation is given by Hooke's Law $\varepsilon_{el} = \sigma/E$, then

$$\varepsilon_{tot} = \frac{\sigma}{E} + \varepsilon_{in}$$

$$\dot{\sigma} = E(\dot{\epsilon}_{tot} - \dot{\epsilon}_{in})$$

During the stress relaxation stage of the test, strain is maintained at a constant value, which implies that total strain rate equals zero.

$$\dot{\epsilon}_{tot} = \frac{\dot{\sigma}}{E} + \dot{\epsilon}_{in} = 0$$

$$\dot{\epsilon}_{in} = -\frac{\dot{\sigma}}{E}$$

Numerical data provided can be plotted in a $\sigma - \sigma_i$ vs. $\dot{\epsilon}_{in}$.

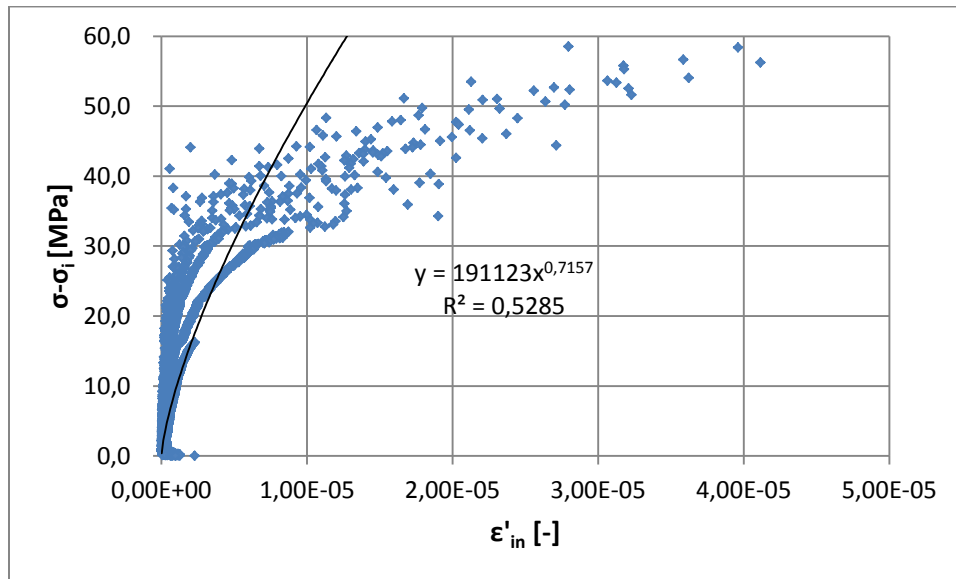


Figure 12 - Viscous effects relaxation @ 20°C

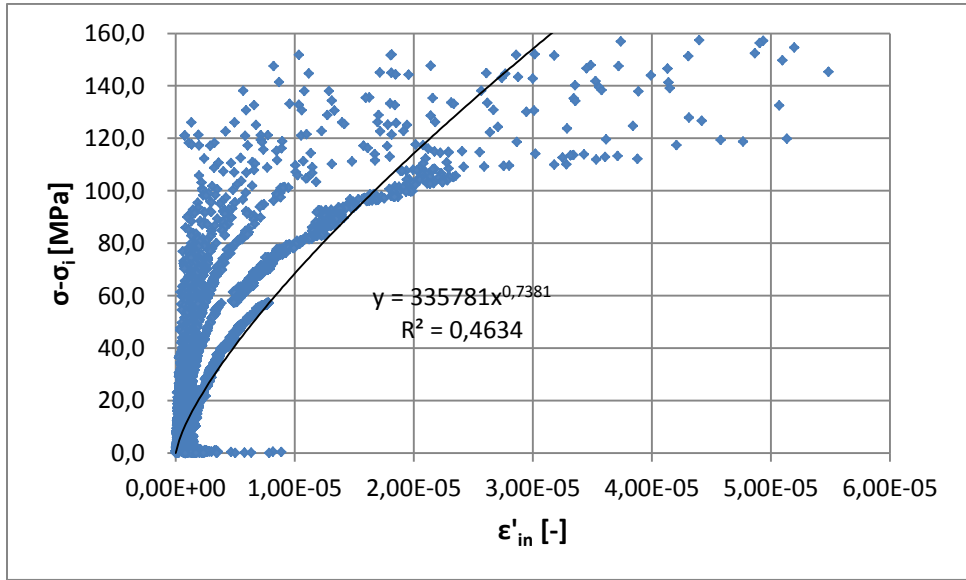


Figure 13 - Viscous effects relaxation @ 500°C

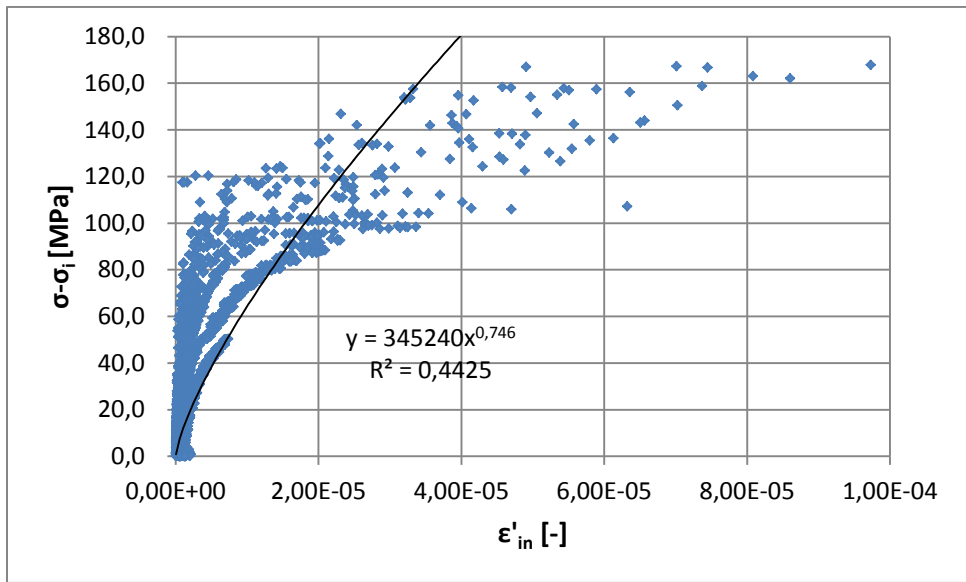


Figure 14 - Viscous effects relaxation @ 600°C

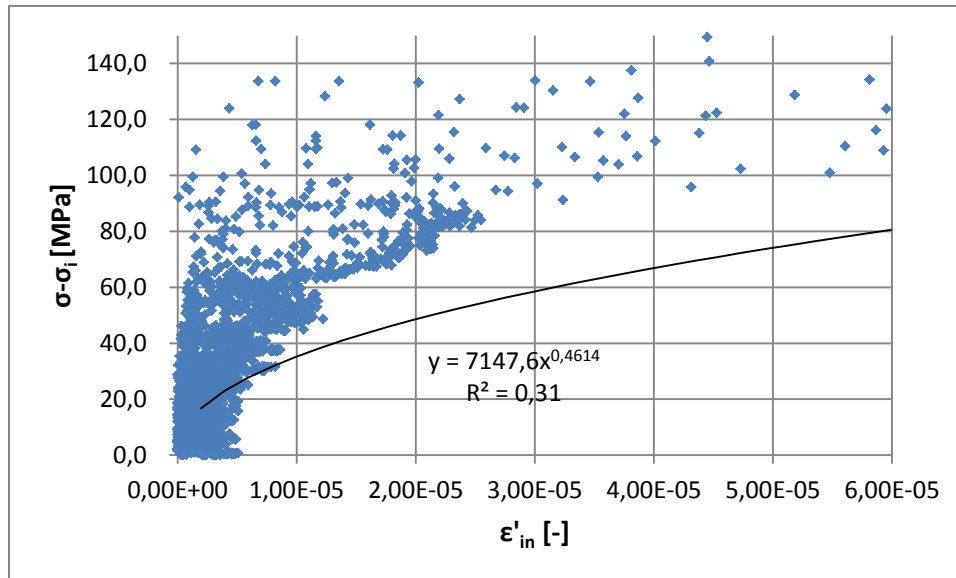


Figure 15 - Viscous effects relaxation @ 700°C

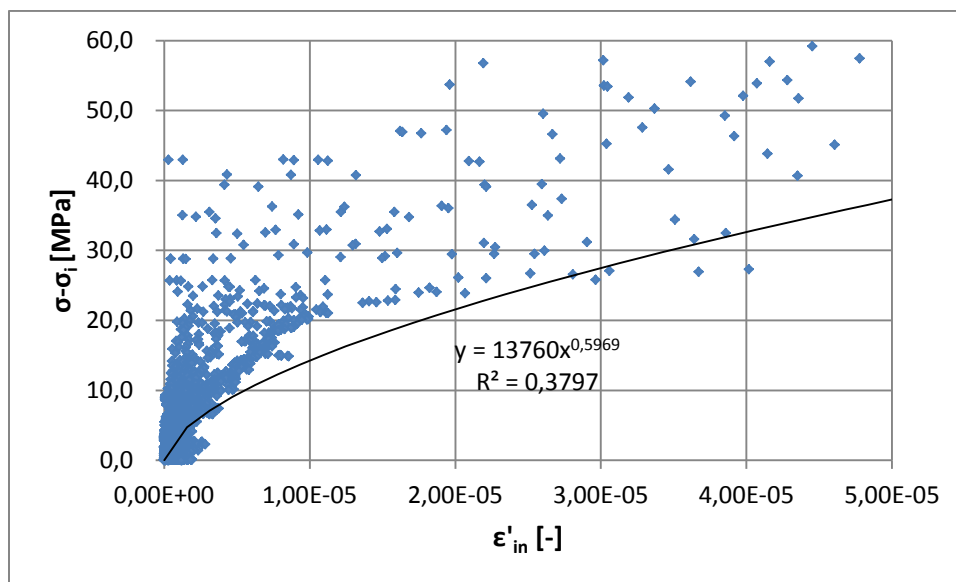


Figure 16 - Viscous effects relaxation @ 800°C

As the experiment progresses, viscous effects relax and the measured stresses approach the final value of σ_i (non-viscous stress). This implies that the points closer to the origin in the graphs correspond to the last stages of the experiment.

It can be observed that the plotted points show a trend that could be assumed exponential, which would adjust to the mathematical model given by:

$$\sigma - \sigma_i = K \epsilon_{in}^{1/n}$$

However, a power law trend line obtained by least squares regression is an extremely poor approximation, as it is shown.

In order to improve the approximation, final values could be omitted, as the stress approaches non-viscous stress value. However, this methodology would yield highly unreliable results, given that there is no clear criterion to define the interval to be removed.

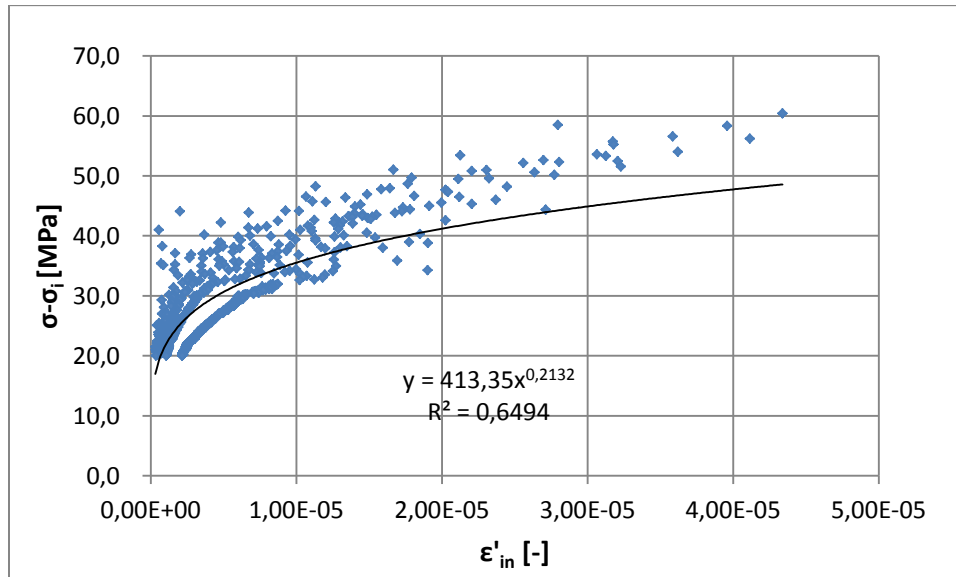


Figure 17 - Viscous effects relaxation (simplified approximation) @ 20°C

Eliminating values in the 30% lower end, the approximation improves, but the error (given by R^2) is still very high.

7. To obtain the relationship between the stress and the inelastic strain rate, the following procedure has to be followed:

Given the tensile test + stress relaxation test:

$$t, \sigma, \epsilon, \sigma_i, E \rightarrow \text{known}$$

As it has demonstrated:

$$\dot{\epsilon}'_{in} = -\frac{\dot{\sigma}}{E}$$

Approximating with finite differences:

$$\dot{\sigma} \approx \frac{\Delta\sigma}{\Delta t} = \frac{\sigma_{i+1} - \sigma_i}{t_{i+1} - t_i}$$

Given the relationship between stress and the final value of non-viscous stress after stress relaxation:

$$\sigma - \sigma_i = K \dot{\epsilon}_{in}^{1/n}$$

$$\log(\sigma - \sigma_i) = \log K + \frac{1}{n} \log \dot{\epsilon}_{in}$$

It should be considered that this relation is valid during the stage of the stress relaxation test, that is to say, once the strain has become constant:

ϵ : constant

Using the linear simplification for the graphs plotted, the following graphs are obtained.

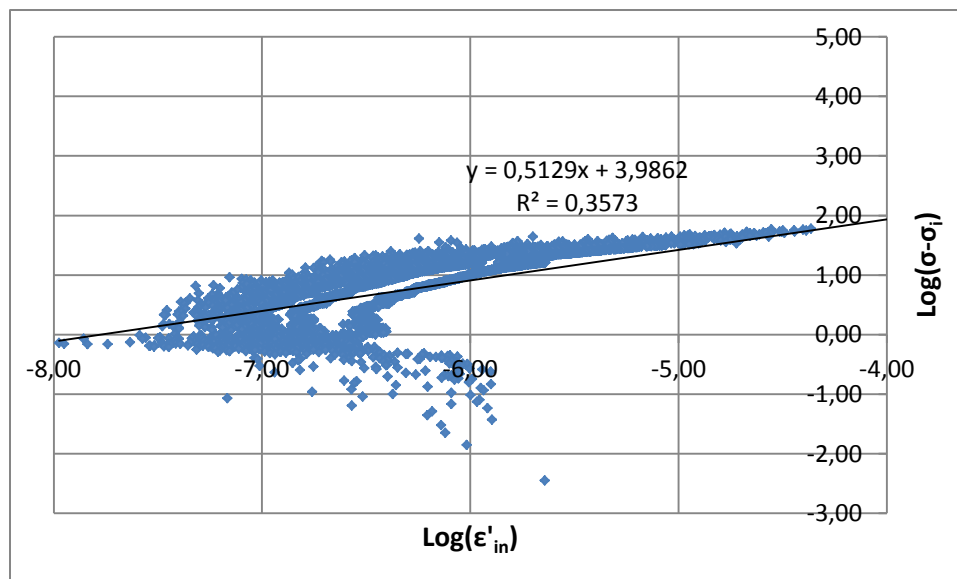


Figure 18 - Linearization graph @ 20°C

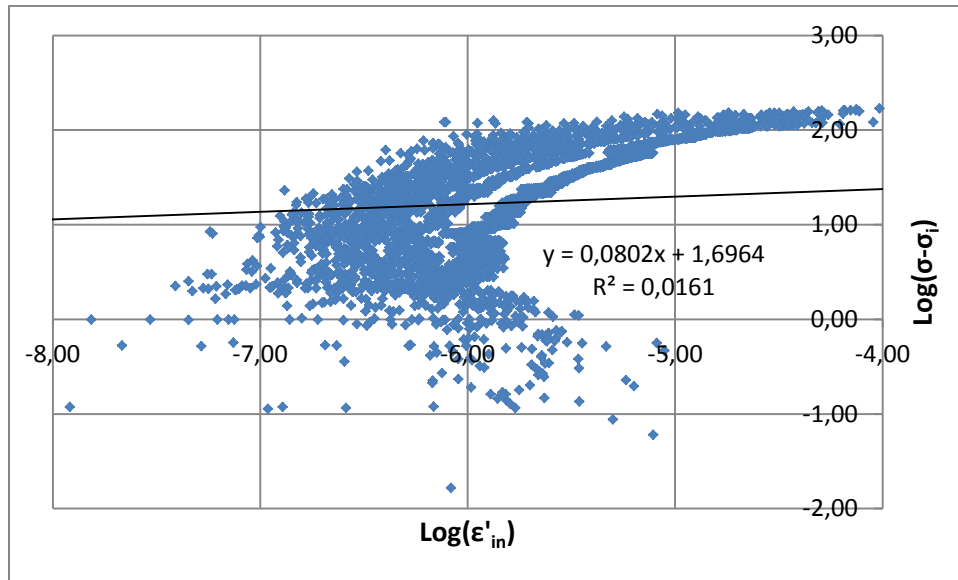


Figure 19 - Linearization graph @ 500°C

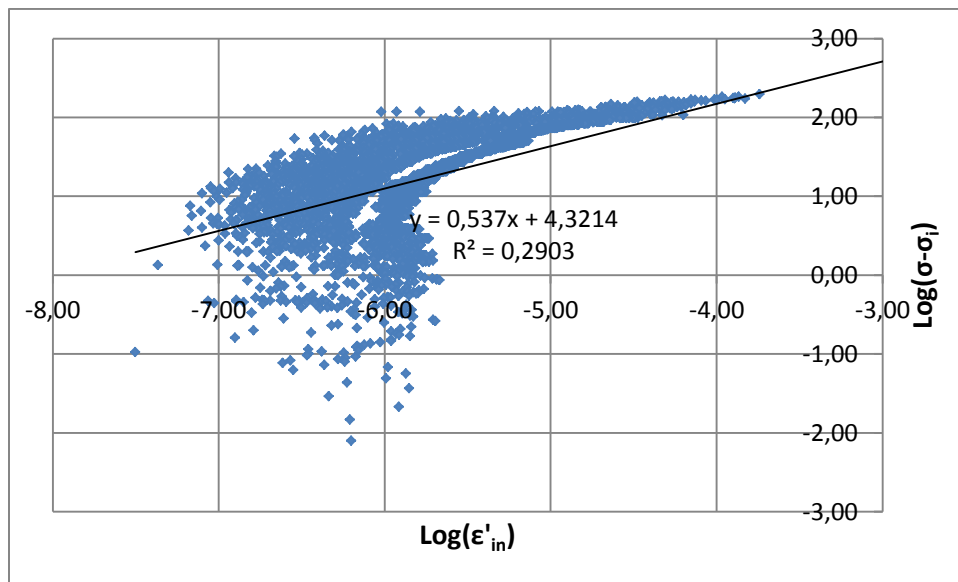


Figure 20 - Linearization graph @ 600°C

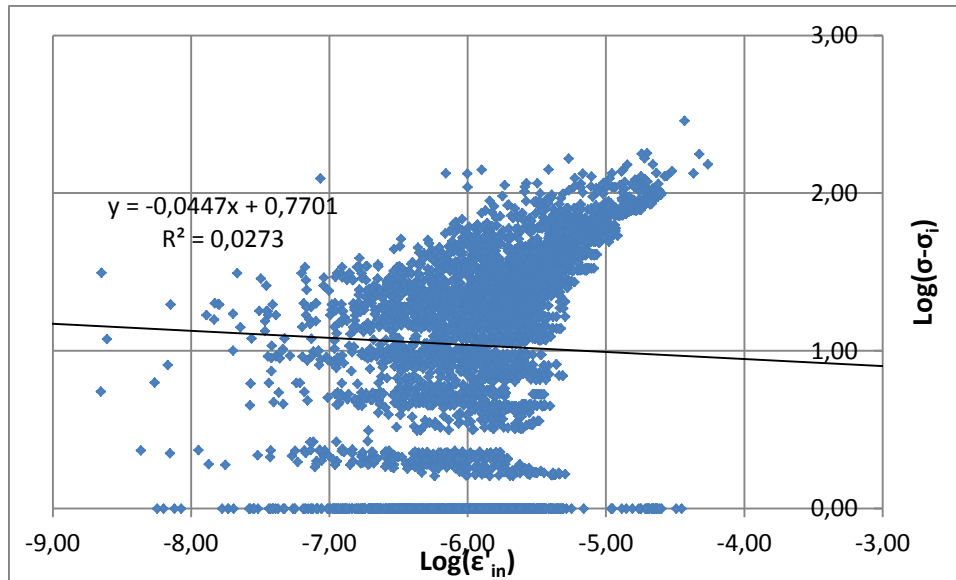


Figure 21 - Linearization graph @ 700°C

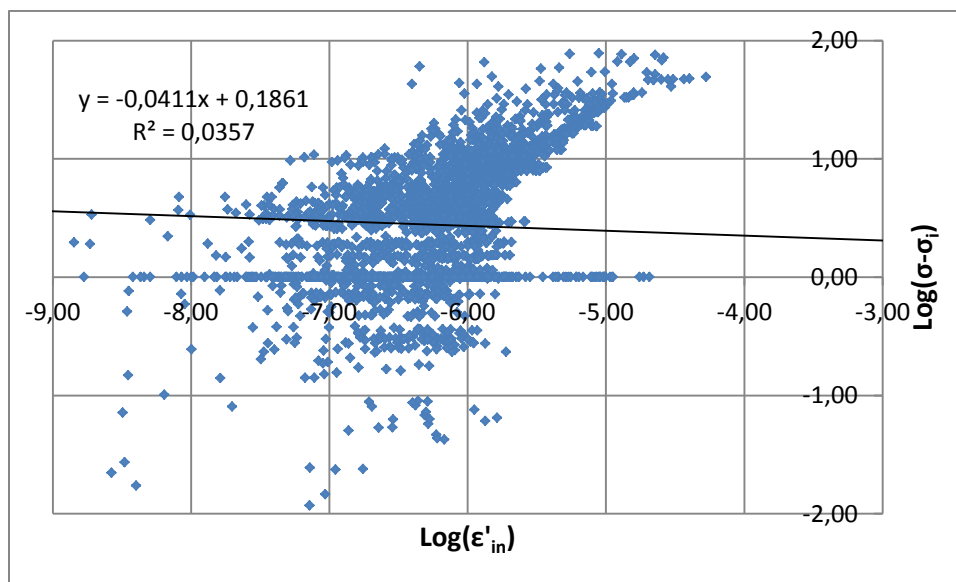


Figure 22 - Linearization graph @ 800°C

As it can be clearly observed, dispersion is too big and trend lines are not useful.

8. If the linearization had yielded any reliable results, a simple procedure could have been applied to obtain the constant values K and n :

Given the equation of the linear approximation: $y = A \cdot \log(x) + B$

$$K = 10^B \quad n = 1/A$$