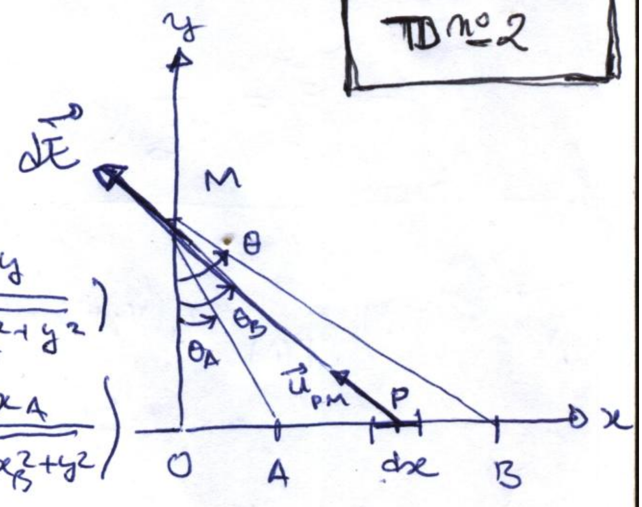
Ex2)

Corrigé du TD n°2

• $\vec{dE} = K \dfrac{\lambda dx}{PM^2} \vec{u}_{PM}$ on a : $tg\theta = \dfrac{x}{y} \Rightarrow \dfrac{d\theta}{\cos^2\theta} = \dfrac{dx}{y}$, $y = PM \cos\theta$

$$\vec{dE} = \frac{k \lambda d\theta}{y} \vec{u}_{PM}$$

$$\vec{dE} \begin{cases} dE_x = -\dfrac{k\lambda}{y} \sin\theta d\theta \Rightarrow E_x = -\dfrac{k\lambda}{y}\displaystyle\int_{\theta_A}^{\theta_B} \sin\theta d\theta \\ dE_y = \dfrac{k\lambda}{y}\cos\theta d\theta \Rightarrow E_y = -\dfrac{k\lambda}{y}\displaystyle\int_{\theta_A}^{\theta_B} \cos\theta d\theta \end{cases}$$



$$\begin{cases} E_x = \dfrac{k\lambda}{y}(\cos\theta_B - \cos\theta_A) = \dfrac{k\lambda}{y}\left(\dfrac{y}{\sqrt{x_B^2+y^2}} - \dfrac{y}{\sqrt{x_A^2+y^2}}\right) \\ E_y = \dfrac{k\lambda}{y}(\sin\theta_B - \sin\theta_A) = \dfrac{k\lambda}{y}\left(\dfrac{x_B}{\sqrt{x_B^2+y^2}} - \dfrac{x_A}{\sqrt{x_B^2+y^2}}\right) \end{cases}$$

• $M \in AB \Rightarrow x_A = -x_B$

$$\begin{cases} E_x = 0 \\ E_y = \dfrac{2k\lambda \, x_B}{y\sqrt{x_B^2+y^2}} \end{cases}$$

• fil ∞ $\quad x_A \to -\infty$, $x_B \to +\infty$ $\quad \begin{cases} E_x = 0 \\ E_y = \dfrac{2k\lambda}{y} \end{cases}$ -

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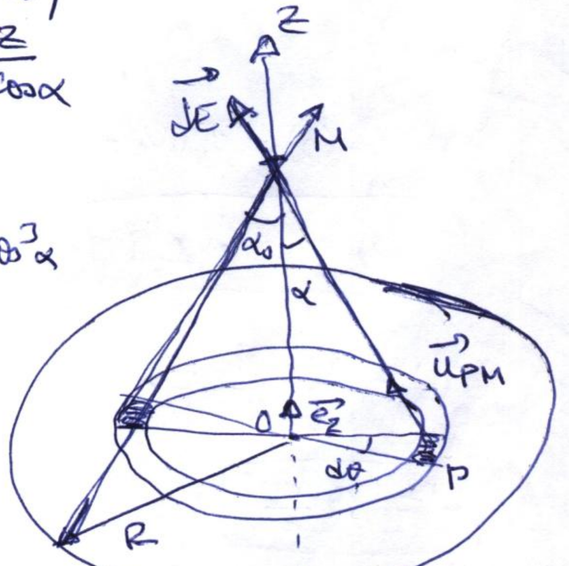
Ex3

$$\vec{d^2E} = k \frac{\sigma ds}{PM^2} \vec{u}_{PM} \qquad (ds = dr\, r d\theta)$$

$$PM = \frac{z}{\cos\alpha}$$

$$\vec{d^2E} = \frac{k r\, dr d\theta}{z^2} \sigma \cos^2\alpha$$

$$dE_z = \frac{k r dr}{z^2} \sigma \cos^3\alpha \int_0^{2\pi} d\theta = \frac{k 2\pi r dr}{z^2} \sigma \cos^3\alpha$$



$$tg\alpha = \frac{r}{z} \Rightarrow \frac{d\alpha}{\cos^2\alpha} = \frac{dr}{z}$$

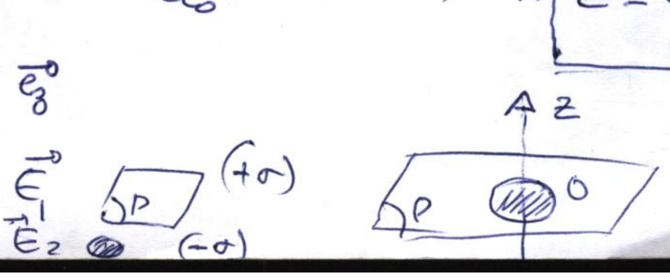
$$dE_z = k \frac{2\pi}{z^2} z (tg\alpha) z \frac{d\alpha}{\cos^2\alpha} \sigma \cos^3\alpha$$

$$= k 2\pi\sigma \sin\alpha d\alpha$$

$$E_z = k 2\pi \sigma \int_0^{\alpha_0} \sin\alpha d\alpha = \frac{\sigma}{2\varepsilon_0}(1 - \cos\alpha_0) \Rightarrow \boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{(z^2+r^2)^{1/2}}\right)\vec{e}_z}$$

• $R \to \infty$ $\quad \vec{E} \to \dfrac{\sigma}{2\varepsilon_0} \vec{e}_z$

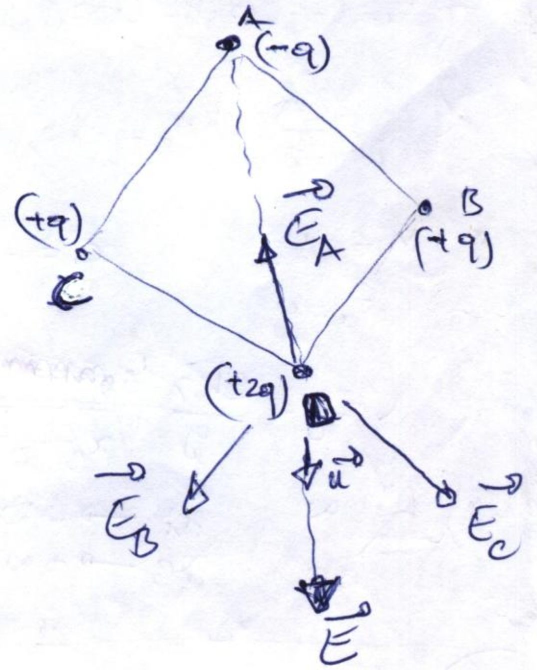
$$\vec{E} = \vec{E}_1 + E_2$$



$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{k} + \left[-\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{(z^2+r^2)^{1/2}}\right]\vec{e_z}\right.$$

$$\Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{(z^2+r^2)^{1/2}}\right)\vec{e_z}.$$

Ex 4



1/ a) $E_{p(A)} = q_A(V_B + V_C + V_D)$

$= -q\left(\frac{kq}{a} + \frac{kq}{a} + \frac{2kq}{a\sqrt{2}}\right)$

$= -\frac{kq^2}{a}\left(2 + \frac{2}{\sqrt{2}}\right)$

$\boxed{E_{p(A)} = -3{,}41\ \frac{kq^2}{a}}$

b) $E_{p(B)} = q_B(V_A + V_C + V_D)$

$= +q\left(-\frac{kq}{a} + \frac{kq}{a\sqrt{2}} + \frac{2kq}{a}\right)$

$= \frac{kq^2}{a}\left(1 + \frac{1}{\sqrt{2}}\right)$

$\boxed{E_{p(B)} = 1{,}7\ \frac{kq^2}{a}}$

c/ $E_{p(C)} = E_p(B) = \boxed{E_p(C) = 1{,}7\ \frac{kq^2}{a}}$

d/ $E_p(D) = q_D(V_A + V_C + V_B) = 2q\left(-\frac{kq}{a\sqrt{2}} + \frac{kq}{a} + \frac{kq}{a}\right) = \frac{2kq^2}{a}\left(2 - \frac{1}{\sqrt{2}}\right)$

$\boxed{E_p(D) = 2{,}6\ \frac{kq^2}{a}}$

2/ $U = \frac{1}{2}\sum_i E_{p_i} = \frac{1}{2}\left(E_p(A) + E_p(B) + E_p(C) + E_p(D)\right) = 1{,}3\ \frac{kq^2}{a}$

$\boxed{U = 1{,}3\ \frac{kq^2}{a}}$

3/ $E_B = E_C = \frac{kq}{a^2}$ $\boxed{E_B = E_C = \frac{kq}{a^2}}$

$E_A = \frac{kq}{(a\sqrt{2})^2} = \frac{kq}{2a^2}$ $\boxed{E_A = \frac{kq}{2a^2}}$